

Dramatis Personae

• **Type** $a: U$: int, string MyClass, MyRecord

• **Function** $F: U \rightarrow V$ $F: \text{int} \rightarrow \text{int}$
• Pure • (no side effects) $F(x) = x+1$ $g(x) = x+2$

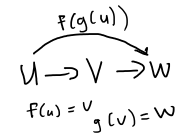
• **Tuples** (U, V) $F: (U, V) \rightarrow W$
ex $F: (\text{int}, \text{int}) \rightarrow \text{int}$
 $f(x, y) = x+y$

Act 1 Categories

• **Types** U

• **Arrows** $U \rightarrow V$ 'Set'

id: $U \rightarrow U$



Composition

associative

ex **Op**: same types
 $U \xrightarrow{\text{op}} V = V \rightarrow U$

equality

$\{1\} \rightarrow \{1\}$
 $2 \rightarrow \{0,1\}$
 $\{3\} \rightarrow \{0,0,0\}$

Counting
'natural' identification
Sets are equal if
natural identification

Currying:

$(U, V) \rightarrow W = U \rightarrow (V \rightarrow W)$
 $F(u, v) = w \Leftrightarrow \bar{F}(u)(v) = w$
partial application

Act 2

Functors

$F \langle U \rangle$

• **map** $F_{\text{map}}: (U \rightarrow V) \rightarrow (F \langle U \rangle \rightarrow F \langle V \rangle)$

• **identity, composition**

$T \langle U \rangle = (U, A)$

$T_{\text{map}}: F: U \rightarrow V \rightarrow \bar{F}: (U, A) \rightarrow (V, A)$

$\bar{F}(u, a) = (F(u), a)$

$H \langle U \rangle = A \rightarrow U$

$H_{\text{map}}: F: U \rightarrow V \rightarrow \hat{F}: A \rightarrow V$

$\hat{F}(g)(a) = F(g(a))$

Currying (redux)

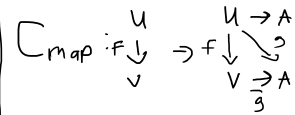
$T \langle U \rangle \rightarrow V = U \rightarrow H \langle V \rangle$

T, H Adjoint Pair

$C^* \langle U \rangle \xrightarrow{\text{op}} V = U \rightarrow C^* \langle V \rangle$

c^*, c^* Adjoint pair

$C^* \langle U \rangle = U \rightarrow A$



$U \rightarrow V \rightarrow A$
 $f \downarrow \quad g \downarrow$
 $U \rightarrow A$
 $f \downarrow \quad g \downarrow$
 $V \rightarrow A$

Contravariant

int \rightarrow dec

width: int \rightarrow div

op \uparrow \downarrow
width: decimal \rightarrow div

Act 3:

Monads:

• **Functor** $M \langle U \rangle$

• **return** $U \rightarrow M \langle U \rangle$

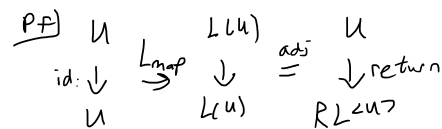
• **bind**: $(U \rightarrow M \langle V \rangle) \rightarrow (M \langle U \rangle \rightarrow M \langle V \rangle)$

+ **compatibility**

Fact (Kleisli):

Monads = Adjoint Pairs

if $R \langle U \rangle \leftarrow L \langle U \rangle \rightarrow V = U \rightarrow R \langle V \rangle$
is a monad



Currying (redux)

T, H

\downarrow

$H \langle U \rangle = A \rightarrow (U, A)$

'State Monad'

obj: A mutated

method (this: A, input: U) = output: V, this: A

\uparrow

$U \rightarrow A \rightarrow (V, A)$

input this output discard this

\downarrow bind

allows us to call
multiple methods
& keep track of state

C^*, c^* adjoint pair

$C \langle U \rangle = (U \rightarrow A) \rightarrow A$
'continuation monad'

$A = \text{Task}; U \rightarrow A$ 'action'

$(U \rightarrow A) \rightarrow A$ is callback!

$F(u, \text{callback}) = \text{void / Task}$

$U \rightarrow (U \rightarrow A) \rightarrow \text{Task}$