Dramatis Personae

- **Type** $u : U$: int, string, MyClass, MyRecord
- **Function** $F : U \to V$ $\quad F : int \to int$ $\quad F(x) = x+1$ $\quad g(x) = x+2$
  - Pure · (no side effects)
- **Tuples** (U, V) $\quad F : (U, V) \to W$
  ex $F : (int, int) \to int$
  $+(x, y) = x + y$

Act 1 Categories

- **Types** U
- **Arrows** $U \to V$ 'set'
  id: $U \to U$
  Composition $U \to V \to W$ $\quad f(g(u))$
  $f(u) = v$ $\quad g(v) = w$
  associative

ex **Op**: same types
$U \xrightarrow{op} V = V \to U$

equality

$\{1 \to \{(\cdot)$
$2 \to (\cdot\cdot)$
$3\} \to (\cdot\cdot\cdot)\}$
counting
'natural' identification
sets are **equal** if
natural identification

Currying: partial application
$(U, V) \to W = U \to (V \to W)$
$F(u, v) = w \leftrightarrow \bar{F}(u)(v) = w$

Act 2

Functors

$F\langle U \rangle$
- **map** $\quad F_{map} : (U \to V)^{set} \to (F\langle U \rangle \to F\langle V \rangle)^{set}$
- **identity, composition**

$T^A\langle U \rangle = (U, A)$

$T_{map} : \begin{matrix} U \\ \downarrow \\ V \end{matrix} \to \begin{matrix} (U,A) \\ \downarrow \bar{F} \\ (V,A) \end{matrix}$

$\bar{F}(u, a) = (F(u), a)$

$H^A\langle U \rangle = A \to U$

$H_{map} : F : \begin{matrix} U \\ \downarrow \\ V \end{matrix} \to \begin{matrix} A \to U \\ \downarrow \bar{F} \\ A \to V \end{matrix}$ $\quad A \xrightarrow{g} U \xrightarrow{F} V$

$\bar{F}(g)(a) = F(g(a))$

Currying (redux)

$T\langle U \rangle \to V = U \to H\langle V \rangle$

T, H Adjoint Pair

$C^A\langle U \rangle \xrightarrow{op} V = U \to C^A\langle V \rangle$

C^*, C^* Adjoint pair

$C^A\langle U \rangle = U \to A$

$C_{map} : f : \begin{matrix} U \\ \downarrow \\ V \end{matrix} \to \begin{matrix} U \to A \\ \downarrow f \\ V \to A \end{matrix}$ $\quad U \to A \xrightarrow{\bar{g}} V \to A$

$f : \begin{matrix} U \\ \downarrow \\ V \end{matrix} \to \begin{matrix} U \to A \\ \downarrow op\,\bar{g} \to \bar{g}(f(a)) \\ V \to A \end{matrix}$

Contravariant

int → dec

width : int → div
op ↑ ↓
width : decimal → div

Act 3

Monads:

- **Functor** $M\langle U \rangle$
- **return** $U \to M\langle U \rangle$
- **bind** : $(U \to M\langle V \rangle) \to (M\langle U \rangle \to M\langle V \rangle)$
- **+ compatibility**

Fact (Kleisli)

Monads = Adjoint Pairs

$RL\langle U \rangle \leftarrow L\langle U \rangle \to V \overset{adj}{=} U \to R\langle V \rangle$ if
$RL\langle U \rangle$ is a monad

Pf $\quad \begin{matrix} U \\ \downarrow id \\ U \end{matrix} \xrightarrow{L_{map}} \begin{matrix} L(U) \\ \downarrow \\ L(U) \end{matrix} \overset{adj}{=} \begin{matrix} U \\ \downarrow return \\ RL\langle U \rangle \end{matrix}$

Currying (redux)

T, H

↓

$HT\langle U \rangle = A \to (U, A)$

'State Monad'

obj : A (mutated)
- method (this: A, input: U) = output: V, this: A

↕

$U \to A \to (V, A)$

input: this; output: discard this

↓ bind

allows us to call multiple methods & keep track of state

C^A, C^A adjoint pair

$CC\langle U \rangle = ((U \to A) \to A)$

'continuation monad'

$A = Task$; $U \to A$ 'action'

$(U \to A) \to A$ is callback!

$F(U, callback) = void\ /\ Task$

$U \to (U \to A) \to Task$